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REWARD FUNCTIONS AND COOPERATIVE GAMES: CHARACTERIZATION AND ECONOMIC APPLICATION

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In this paper we study network structures in which the possibilities for cooperation are restricted and the benefits of a group of players depend on how these players are internally connected. One way to represent this type of situations is the so-called reward function, which represents the profits obtainable by the total coalition if links can be used to coordinate agents' actions. For any cooperative game, a reward function is associated. Given a reward function, our aim is to analyze under which conditions it is possible to associate a cooperative game to it. We characterize the reward function by means of two conditions, component permanence and component additivity, in order to determine whether there exists or not a cooperative game associated to it. An economic application is shown to illustrate the main theoretical result. Data from Catalan firms is used to compute the reward function on the set of communication networks determined by firms, customers, distributors and suppliers.

Keywords: Cooperative game; network; reward function.

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1. Introduction

The concept of network has been applied in many scientific and social contexts. For example, from the point of view of business organization, Boorman (1975) and

Keren and Levhari (1983) have made contributions in the field of microeconomy, in which they used network structures to study the internal organization of businesses.

Nowadays, there is a renewed interest for the analysis of networks in the context of an economy based on knowledge and on the intensive use of information and communication technologies (ICT). The evolution of business towards networked business implies the decentralization of its activities and moves to a structure based on the interconnections in networks of all the elements of the value chain (Torrent and Vilaseca (2004)).

Game Theory facilitates tools to analyze network structures. Usually, the techniques used in the literature to represent these types of connections are the communication networks. Myerson (1977) was the precursor of graphic representation of situations where the possibilities for cooperation are restricted. For a summary of this type of situations, we refer to Slikker and van den Nouweland (2001). Borm *et al.* (1992) carried out an approximation using the so-called link games based on the connections rather than on the players. In the same research line, Jackson and Wolinsky (1996) constructed the reward function, which they named value function, that is differentiated from the previous approximation mainly in that the profits of a group of players depend more on how these players are internally connected. Given a cooperative game, a reward function can be associated in a natural way. Our purpose is to study the inverse implication.

The paper presents a main theoretic result. The main contribution of the paper to the existing literature is the characterization of the reward function that allows us to determine when it has or not a cooperative game associated. It proves that given a function on the set of all network structures to the real numbers, the component permanence and the component additivity are necessary and sufficient conditions for the existence of a unique cooperative game (with individual values of zero) that has associated as a reward function precisely the initial function.

As a consequence of the theorem, it is proved that component additivity is also a necessary and sufficient condition for the cooperative game being 0-normalized.

The theoretical framework of this result will permit us to present an interesting economic illustration of the characterization theorem of the reward function. Specifically, the application is based on data extracted from Catalan firms, from which the reward function is generated for the set of possible networks. This reward function will measure the degree of openness of the economy when ICT are used in cooperation. The networks consist of four nodes/players (firms, customers, distributors and suppliers) and we consider the connection between a pair of nodes to exist when information and communication technologies are used in communication.

The structure of the paper is as follows. In Sec. 2 we recall some basic game theoretic notions and we provide some necessary definitions and concepts about the Theory of Networks. In Sec. 3 we give the main result of the paper, the characterization of the reward function. An economic application of the theoretical result with the corresponding analysis from the perspective of Game Theory is shown in Sec. 4. And finally, in Sec. 5 we conclude with some final remarks.

2. Notation and Preliminaries

2.1. *Cooperative games and solution concepts*

For the purpose of the paper, we will formalize those situations in which different agents cooperate to achieve a common objective. We will specifically model these situations via Cooperative Game Theory.

A cooperative game with transferable utility or TU-game is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of players and v is the characteristic function that assigns to every coalition $S \subseteq N$ of players a value $v(S) \in \mathbb{R}$, representing the profits or the value that coalition S can obtain if the players of S cooperate to achieve a common objective, with independence from the actions of the players who do not belong to the coalition. By convention $v(\emptyset) = 0$. The concept of transferable utility refers to the fact of assuming that the utility can be transferred freely between players, taking into account that the measure of this utility is common and infinitely divisible.

In certain situations we may want to work with games that have individual values of zero, which is to say $v(\{i\}) = 0$ for all $\{i\} \in N$, since we can homogenize the problem and even simplify the reasoning. Games with individual values of zero are called 0-normalized games. To simplify notation, from now on we write i instead of $\{i\}$ to refer to player i.

In cooperative games it is customary to assume that players cooperate and form the grand coalition, in this sense, Game Theory tries to supply solution concepts that distribute the benefits or values among the different players. An allocation or payoff vector is a vector $x = (x_i)_{i \in N} \in \mathbb{R}^n$ that assigns to each player $i \in N$ the payoff or profit x_i that can be obtained if he cooperates with the other players.

One of the most well-known single-valued solution concepts for its axiomatic properties is the Shapley value. The Shapley value was introduced by Shapley (1953) and is based on the concept of marginal contribution. Given a cooperative game (N, v) the Shapley value $\phi(v) \in \mathbb{R}^n$ is defined as:

$$\phi_i(v) = \sum_{S \subseteq N \setminus i} \frac{s!(n-s-1)!}{n!}(v(S \cup i) - v(S)), \quad i = 1, 2, \ldots, n.$$

2.2. *Networks*

While in the previous section we assumed that all coalitions of players could be formed, in certain situations this is not the case. In this section, we will study these types of situations due to the fact that our objective is to relate situations of cooperation with structures in network.

Given a set of players $N = \{1, 2, \ldots, n\}$, in order to be able to coordinate their actions they have to be able to communicate. We will represent the bilateral channels of communication between players via a communication network, that is to say, by a graph (N, L) where $N = \{1, 2, \ldots, n\}$ is the set of players that we will situate in the vertices or nodes of the graph and where these players are connected by a set of arcs or links $L \subseteq L^N = \{\{i, j\} | i, j \in N, i \neq j\}$. The connection

$\{i, j\}$ indicates that the players i and j can communicate without requiring the intermediation of other players, i.e., i and j are directly connected in the network. In general, we say that i and j are connected in the network if there is a path that joins them, in other words, if there is a sequence of players $(i_1, i_2, \ldots, i_t)$ such that $i_1 = i$, $i_t = j$ and $\{i_k, i_{k+1}\} \in L$ for all $k \in \{1, 2, \ldots, t-1\}$. If two players are connected, but are not directly connected, we say that they are indirectly connected in the network. A network (N, L) is complete if all pairs of players are directly connected in the network, i.e., $L = L^N$.

Given a coalition $S \subseteq N$ where $|S| = 2, 3, \ldots, n$, we denote with L_S the structures of links that result in complete networks. $L_S = \{\{i, j\} | i, j \in S\}$ is a structure of links where the players of S are all directly connected and the rest of players are not connected.

If there are players who are not connected in a network, this means that they cannot communicate and, therefore, that we can divide the set of players N into components of cooperation. Thus, we say that i and j are in the same component C if and only if they are connected, either directly or indirectly. The set of components of the network (N, L) will be denoted as N/L.

Given $C \in N/L$, we will define $L(C) = \{\{i, j\} \in L | i, j \in C\}$. So, $L(C)$ is the network associated to the component C where there are only the links of C and where the individual players $k \in N \backslash C$ are not connected with anyone. In this way, for example, if we consider the network (N, L) where $N = \{1, 2, 3, 4, 5\}$ and $L = \{\{1, 2\}, \{1, 3\}, \{4, 5\}\}$, then the set of components of (N, L) is $N/L = \{\{1, 2, 3\}, \{4, 5\}\}$ and, for example, the network associated to component $C = \{1, 2, 3\}$ is $L(\{1, 2, 3\}) = \{\{1, 2\}, \{1, 3\}\}$.

3. Characterization of the Reward Function

There are certain situations in which the economic possibilities of the players cannot be directly represented via cooperative games. Next, we will present a model, Slikker and van den Nouweland (2001), in which the profits of a group of players does not only depend on their connected components but also on the internal structure of these components, in other words, on how players are connected.

As in the case of link games (Borm *et al.* (1992)), the representation of these situations cannot be described via the characteristic function, in these cases the so-called reward function is constructed. The reward function is defined as a function that assigns a real value $r(L)$ to each set of links $L \subseteq L^N$ that represents the profits obtainable by the grand coalition in network (N, L) if the connections of L can be used to coordinate players' actions.

For any cooperative game (N, v), a reward function r is associated in the following way:

$$r(L) = \sum_{C \in N/L} v(C) \quad \text{for all } L \subseteq L^N. \tag{1}$$

For this reward function two properties can be defined.

Definition 1. Given a set of players N and a reward function r defined on subsets of L^N, the reward function is component permanent if it holds that:

$$r(L_1) = r(L_2) \quad \text{for all } L_1, L_2 \subseteq L^N \text{ such that } N/L_1 = N/L_2. \tag{2}$$

Definition 2. Given a set of players N and a reward function r defined on subsets of L^N, the reward function is component additive if it holds that:

$$r(L) = \sum_{C \in N/L} r\left(L(C)\right) \quad \text{for all } L \subseteq L^N. \tag{3}$$

Otherwise, we say that the reward function presents externalities.

In this section, we will focus on the characterization of the reward function. Precisely, what we will prove is that given a function, the component permanence and the component additivity are necessary and sufficient conditions for the existence of a unique 0-normalized associated cooperative game on player set N that has as its reward function precisely this function.

Next, we will see that if the reward function does not generate externalities, then the value of this function for the case with no connection between nodes is zero.

Lemma 1. *Given a set of players $N = \{1, 2, \ldots, n\}$ such that $|N| \geq 2$ and a reward function $r : 2^{L^N} \to \mathbb{R}$ such that r is component additive, then $r(\emptyset) = 0$.*

Proof. Since r is component additive, it satisfies $r(L) = \sum_{C \in N/L} r(L(C))$ and, therefore:

$$r(\emptyset) = \sum_{i \in N} r(L(i)) = \sum_{i \in N} r(\emptyset) \Rightarrow r(\emptyset) = n \cdot r(\emptyset) \Rightarrow r(\emptyset) = 0.$$

□

The following theorem characterizes the reward function.

Theorem 1. *Given a 0-normalized cooperative game (N, v), the reward function r of the game (1) satisfies component permanence (2) and component additivity (3).*

Conversely, given a function $r : 2^{L^N} \to \mathbb{R}$ satisfying (2) and (3), there exists a unique 0-normalized game (N, v) such that its reward function is r.

Proof.

$\Rightarrow$) We have to prove that given a 0-normalized cooperative game (N, v), the reward function $r : 2^{L^N} \to \mathbb{R}$ associated to v satisfies (2) and (3):

1. Given $L_1, L_2 \subseteq L^N$ such that $N/L_1 = N/L_2$ and given a cooperative game v, if we associate the reward function defined in (1), then it satisfies that $r(L_1) = \sum_{C \in N/L_1} v(C) = \sum_{C \in N/L_2} v(C) = r(L_2)$, so r is component permanent.

2. Without loss of generality, we consider the following structure of components $N/L = \{C_1, C_2, \ldots, C_k\}$. Then according to expression (1) of the reward function associated to v, we have:

$$r(L) = \sum_{C \in N/L} v(C) = v(C_1) + v(C_2) + \cdots + v(C_k). \tag{4}$$

From expression (1) and the 0-normalization of v, the value for a structure of links associated to a single component C_i for all $i \in \{1, 2, \ldots, k\}$ is:

$$r(L(C_i)) = \sum_{C \in N/L(C_i)} v(C) = v(C_i) + \sum_{j \in N \setminus C_i} v(j) = v(C_i).$$

Then, substituting in (4) we obtain:

$$\begin{aligned} r(L) &= v(C_1) + v(C_2) + \cdots + v(C_k) \\ &= r(L(C_1)) + r(L(C_2)) + \cdots + r(L(C_k)) \\ &= \sum_{i=1}^{k} r(L(C_i)) = \sum_{C \in N/L} r(L(C)). \end{aligned}$$

Hence, we can affirm that r is component additive.

$\Leftarrow$) We have to prove that given a function $r : 2^{L^N} \to \mathbb{R}$ such that it satisfies (2) and (3), then a unique 0-normalized cooperative game v exists such that its reward function is r.

We will divide the proof in two steps. First we will prove the existence of the game and next the uniqueness of it.

To show existence we will prove that there exists a 0-normalized cooperative game v that satisfies the system of equations described by (1).

Consider the following cooperative game where $v(i) = 0$ for all $i \in N$ and $v(S) = r(L_S)$ for all $S \subseteq N$, $|S| \geq 2$.

Consider $L \subseteq L^N$ such that L has one component that is not a singleton, and that is formed by players in coalition S. Then by 0-normalization of v, and component permanence of r, it holds that

$$r(L) = r(L_S) = v(S) = v(S) + \sum_{j \in N \setminus S} v(j).$$

Then, for such types of networks, v satisfies (1).

Similarly, consider now $L \subseteq L^N$ such that L has more than one component that are not singletons, and let these components be formed by coalitions $S_1, S_2, \ldots, S_k$. Then by 0-normalization of v, and component permanence and additivity of r, it holds that

$$r(L) = \sum_{i=1}^{k} r(L_{S_i}) = \sum_{i=1}^{k} v(S_i) = \sum_{i=1}^{k} v(S_i) + \sum_{j \in N \setminus \{S_1 \cup S_2 \cup \cdots \cup S_k\}} v(j).$$

Hence, v also satisfies condition (1).

Last, the case $L = \{\emptyset\}$ follows directly from Lemma 1.

Finally, to show uniqueness, suppose that there exist two 0-normalized cooperative games v_1 and v_2 which are solution of the equations system described by (1). Then there exists a coalition $S \subset N$, $|S| \geq 2$, such that $v_1(S) \neq v_2(S)$. Then by (1) and 0-normalization of v_1 and v_2, it holds that:

$$r(L(S)) = v_1(S) + \sum_{j \in N \setminus S} v_1(j) = v_1(S),$$

$$r(L(S)) = v_2(S) + \sum_{j \in N \setminus S} v_2(j) = v_2(S).$$

Which leads to a contradiction, because we had supposed that $v_1(S) \neq v_2(S)$. □

The following corollary tells us that the 0-normalization is a necessary and sufficient condition for the reward function to be component additive.

Corollary 1. *Given a cooperative game (N, v) and $r : 2^{L^N} \to \mathbb{R}$ its reward function, then (N, v) is 0-normalized if and only if r is component additive.*

Proof.

⇒) According to the proof of Theorem 1, if a game v is 0-normalized then its reward function r is component additive.

⇐) If v is not 0-normalized, then $\sum_{j \in N} v(j) \neq 0 \Rightarrow r(\emptyset) = \sum_{j \in N} v(j) \neq 0$, applying Lemma 1 we can affirm that r is not component additive. Contradiction. □

4. Economic Application: Influence of the Cooperation Through ICT on the Degree of Openness of an Economy

In this section we will show an economic application of the main theorem presented in the previous section.

In many social or economic situations communication takes place through networks, such as telecommunications or trade relationships to name a few. Another possible application that could be represented by means of networks is the new way of doing business, based on the use of ICT in the generation of value, what is known as e-business (Torrent and Vilaseca (2004)).

In this sense it could be very interesting to study in terms of networks structure the possibilities of external connection of firms with those elements of the value chain (Porter (1985)) that do not form part of the internal organizational structure of firms. In this economic application, we will focus on the analysis of these network structures that can be formed via ICT connections between

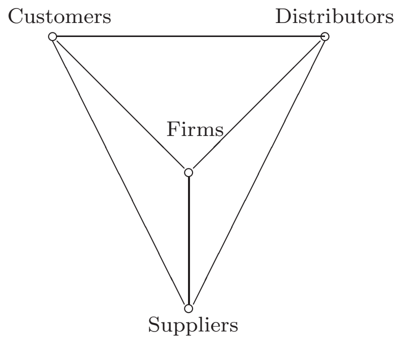

Fig. 1. Connections structure.

firms, customers, distributors and suppliers. Graphically, the network situation that we will analyze is represented in Fig. 1.

With the aim of realizing a practical application of the main theoretical result of the paper, we will take data, obtained from the research project "The information and communication technologies and transformations in Catalan firms",[a] on the percentage of sales in Catalonia, as well as data on the use firms make of ICT in order to connect with its environment.

These data will allow us to obtain each one of the networks structures, as well as the value of the reward function for each structure, that will correspond with the degree of openness of the economy in this situation (step 1). We will see that the fact of incorporating the use of ICT in communications between firms and the aforementioned elements of the value chain, generates an increase in the degree of openness of the economy. From here, if conditions of component permanence and component additivity of Theorem 1 are satisfied, we will be able to construct a cooperative game (step 2) from which we can then analyze the weight of each agent in relation to the influence of the use of ICT in the increase in the degree of openness of Catalan firms (step 3).

Step 1: When assigning values to the set of connections we have only considered connections that include the firm. This is because the other connections do not contribute anything, to generate value it is necessary the firm to be involved, otherwise, in this analysis the connection between the different elements of the value chain such as, for example, distributors and suppliers, would be meaningless. Moreover, we will consider that those connections that give rise to the same component will contribute the same value. Economically this condition makes sense since the circulation of the profit or information is free if there exist connections that allow for this (Castells (2001)).

[a]This study is the PIC-firms project, framed within the Catalonia Internet Project and carried out by the Internet Interdisciplinary Institute of the *Universitat Oberta de Catalunya* with the support of the Generalitat de Catalunya (Vilaseca *et al.* (2003)). http:www.uoc.edu/in3/pic.

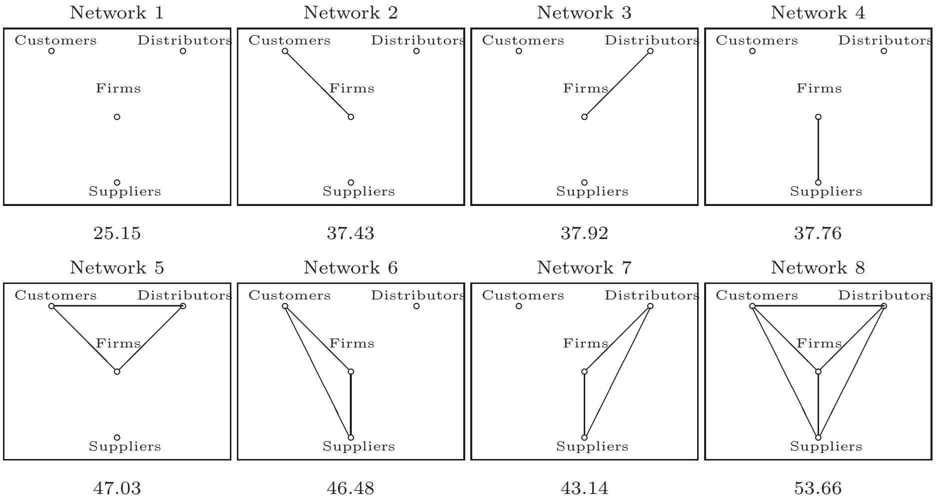

Fig. 2. Complete networks and openness degree of firms.

Figure 2 shows the network structures with the values of the respective degrees of openness of Catalan firms, measured as the percentage of sales outside Catalonia with respect to total sales.

These values allow us to construct the reward function. The reward function will indicate the increments on the degrees of openness that is generated by the use of ICT with respect to the case where ICT are not used in any communication (see network 1 in Fig. 2). Thus, normalizing with respect to the case that there is no connection with ICT, i.e. $L = \{\emptyset\}$, we have the following values for the reward function for all $L \subseteq L^N$. Without loss of generality, from now on we will call firms as agent 1, customers as agent 2, distributors as agent 3 and suppliers as agent 4.

$$r(L) = \begin{cases} 0 & \text{if } 1 \notin L \text{ or } L = \{\emptyset\} \\ 12.28 & \text{if } L = \{1,2\} \text{ or } L = \{\{1,2\},\{3,4\}\} \\ 12.77 & \text{if } L = \{1,3\} \text{ or } L = \{\{1,3\},\{2,4\}\} \\ 12.61 & \text{if } L = \{1,4\} \text{ or } L = \{\{1,4\},\{2,3\}\} \\ 21.88 & \text{if } L = \{\{1,2\},\{1,3\}\} \text{ or } L = \{\{1,2\},\{2,3\}\} \text{ or} \\ & \quad L = \{\{1,3\},\{2,3\}\} \text{ or } L = \{\{1,2\},\{1,3\},\{2,3\}\} \\ 21.33 & \text{if } L = \{\{1,2\},\{1,4\}\} \text{ or } L = \{\{1,2\},\{2,4\}\} \text{ or} \\ & \quad L = \{\{1,4\},\{2,4\}\} \text{ or } L = \{\{1,2\},\{1,4\},\{2,4\}\} \\ 17.99 & \text{if } L = \{\{1,3\},\{1,4\}\} \text{ or } L = \{\{1,3\},\{3,4\}\} \text{ or} \\ & \quad L = \{\{1,4\},\{3,4\}\} \text{ or } L = \{\{1,3\},\{1,4\},\{3,4\}\} \\ 28.51 & \text{otherwise} \end{cases}$$

Step 2: Once the reward function has been obtained, it is easy to check that by construction, our reward function satisfies condition (2) of component permanence.

Also component additivity (3) is straightforward in this case. We can see that externalities are not being generated, since the values of the reward function for those components that do not contain firms are zero due to the fact that firms are veto players. So conditions of Theorem 1 are satisfied and we can associate a cooperative game to our reward function.

Thus, to construct the cooperative game we only have to solve the system of equations generated by expression (1). Following the proof of Theorem 1, the solution of the system is immediate. Specifically, the value of the game for those coalitions that contain firms coincides with the value of the reward function the connections of which generate a set of components among which is a component that coincides with the coalition being considered, while for those coalitions that do not contain firms the value of the game is zero.

So, the cooperative game (N, v), where $N = \{1, 2, 3, 4\}$, associated to the reward function has as its characteristic function:

$$\begin{array}{llll}
v(\emptyset) = 0 & v(\{4\}) = 0 & v(\{2,3\}) = 0 & v(\{1,2,4\}) = 21.33 \\
v(\{1\}) = 0 & v(\{1,2\}) = 12.28 & v(\{2,4\}) = 0 & v(\{1,3,4\}) = 17.99 \\
v(\{2\}) = 0 & v(\{1,3\}) = 12.77 & v(\{3,4\}) = 0 & v(\{2,3,4\}) = 0 \\
v(\{3\}) = 0 & v(\{1,4\}) = 12.61 & v(\{1,2,3\}) = 21.88 & v(\{1,2,3,4\}) = 28.51
\end{array}$$

In this context, we will interpret the value of a coalition S as the contribution of that component to the degree of openness of the economy when its players are cooperating through ICT compared with the absence of the use of ICT in cooperation.

Step 3: Once we have formalized the situation from the slant of Cooperative Game Theory, a natural extension could be to proceed to use this tool to obtain solutions that suggest to us a distribution of the value of the game between the different agents, in order to study the weight of each agent in the degree of openness of the Catalan economy.

One of the single-valued allocations that can be calculated is the well-known Shapley value. The computation of the Shapley value gives the following result:

$$\phi(v) = (15.3658, 5.1392, 4.1075, 3.8975).$$

According to this solution concept, it can be seen that firms are the element that has the greatest weight in the explanation of the increase in sales generated by the use of ICT. This high percentage is in accordance with the fundamental role played by firms. In second place are the customers, who are the next most important element in interpreting the openness of the economy. Distributors occupy third place and suppliers are in last place.

5. Conclusions

The main theoretical result of the paper is the characterization of the reward function. The reward function is a function that assigns to each communication network made up of players and links, a value that indicates the benefit or profit that the grand coalition obtains in that situation. Given a cooperative game, we can naturally associate a reward function to it, which indicates the value that the grand coalition would obtain if only links in the network are possible, that is to say, if there are restrictions in the communication between all the elements of the grand coalition. However, it is not always possible to assign a cooperative game to all reward functions. In this sense, the characterization of the reward function allows us to say when it is possible to associate to it a unique cooperative game. Specifically, for the reward function to have a unique 0-normalized cooperative game assigned, it has to satisfy two conditions: the condition of component permanence and the condition of component additivity. From that main result, we can deduce that given a cooperative game and its reward function, the game is 0-normalized if and only if the reward function is component additive.

Once we have the theoretical tool, we illustrate the main theorem with an economic example, that will study how influences the use of ICT in the communication between different agents of the value chain in the degree of openness of an economy, specifically Catalan economy. The empirical case uses data from a survey done to a representative sample of firms of the Catalan economy. The reward function is generated from this data and once the reward function has been constructed and it has been checked that it satisfies the properties of the theorem, we can proceed to generate the associated cooperative game. The allocation, that distributes the value of the cooperation between all the agents and assigns the weight for each one in the increase of the degree of openness, tells us that firms are the economic agent with the greatest contribution, followed by customers, and then by distributors and suppliers.

With respect to the theoretical results, future research lines are the analysis of conditions that must be satisfied for the reward function to have a unique associated cooperative game, without the restriction of having individual values of zero. Another open question is which conditions must be satisfied to assign a unique cooperative game to the reward function when introducing costs for forming links.

From the empirical side and following the research line focused on costs for establishing links, another practical application to carry out is, for instance, the study of how the use of ICT affects different outputs when the relations of connection involve costs. Another future applications are the analysis of the relations between the different internal components of a firm, such as the area of human resources, infrastructures and innovation, and the generation of value represented by the fact of using ICT in these connections as compared to not using them.

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